Pondicherry university Theory examination -FEB 2023

Fifth semester

Mechanical Engineering

MECHANICAL MEASUREMENTS

2 marks

1. **Define Errors and its types.**

   Error is the difference between the measured values (Vm) and the True value (Vt) of a physical quantity.

   Types of error

   (i)Systematic errors or fixed errors (ii)Errors due to calibration (iii)Human errors (iv)Error of technique (v)Loading error (vi)Instrumentation errors (vii)Illegitimate errors, (viii) Chaotic errors, (ix)Random errors (x)Environmental error

2. **Define Signal conditioning systems.**

   A signal conditioner is a device that modifies raw analog output signals produced by sensors and provide the essential circuitry between the sensor and the data acquisition system.

3. **Mention any two basic methods of Force Measurements.**

   Elastic force meter

   Proving Ring

   Loadcells,

   Strain gauge load cells,

   Hydraulic load cells,

   Pneumatic load cell.

4. **How temperature effects will be handled and compensation may be provided?**

   Temperature compensation is a correction applied to a measurement instrument to reduce errors attributed to temperature changes in a process media which is being measured or in the surrounding environment that the instrument being used.

5. **Classify the various sources of dynamic pressure.**

Submersible pump, Axial pump and Fire hydrant systems.

6. **Define primary and secondary measurement.**

   The measurement obtained directly by observation and comparison is called primary measurement.

   The measurement needs one conversion is called secondary measurement.

7. **Classify various accelerometer calibration methods.**

   A simple calibrator

   Force and impedance measurements

   Logarithmic scales and decibels.

8. **Define Shakers or exciters.**

   A device that produces excitation, in particular a device that provides a magnetizing current for the electromagnets in a motor.

9. **Define Interfacing.**

   Interfacing can be defined as transferring data between microcontrollers and interfacing peripherals such as sensors, analog to digital converters etc.

10. **Define quantization error.**

    It is defined as the difference between the actual analog input value and the nearest quantization level.

11 marks

11. **Explain the various terminologies of measurements**.

    Accuracy: The difference between the measured and true values. Typically, a manufacturer will specify a maximum error as the accuracy; manufacturers often neglect to report the odds that an error will not exceed this maximum value.

    Precision: The difference between the instrument's reported values during repeated measurements of the same quantity. Typically, this value is determined by statistical analysis of repeated measurements.

    Drift: If an instrument does not reproduce the same reading at different times of measurement for the same input signal, it is said to have drift. If an instrument has perfect reproducibility it is said to have no drift.

Stability: A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A **stable system** produces a bounded output for a given bounded input.

This is the response of first order control system for unit step input. This response has the values between 0 and 1.

Linearity: **Linearity** describes that you can describe the effects of a **system** by separating the input signal into simple parts and using superposition at the output to restore the overall **system** output

Error: Error is the difference between the measured values(Vm) and the True value (Vt) of a physical quantity

Resolution: The smallest increment of change in the measured value that can be determined from the instrument's readout scale. The resolution is often on the same order as the precision; sometimes it is smaller.

Sensitivity: The change of an instrument or transducer's output per unit change in the measured quantity. A more sensitive instrument's reading changes significantly in response to smaller changes in the measured quantity. Typically, an instrument with higher sensitivity will also have finer resolution, better precision, and higher accuracy.

Dead Zone: A deadband or **dead-band** (also known as a **dead zone** or a neutral **zone**) is a **band** of input values in the domain of a transfer function in a **control system** or signal processing **system** where the output is zero (the output is '**dead**' - no action occurs).

12. Discuss briefly about various types of errors.

ERROR:

Error is the difference between the measured value (Vm) and the true value (Vt) of a physical quantity. The accuracy of a measurement system is measured in terms of error.

Static Error =Vm-Vt

Error may be positive or negative. If the instrument reads higher than the true value, it is called as positive error and if the instrument reads lower than the true value, it is called as negative error.

A study of error helps in reducing them and helps in finding the reliability of the results.

TYPES OF ERRORS:

Important errors have been listed below.

a. Systematic errors or fixed errors

Errors due to calibration.

Human errors (observation errors and operational errors).

Loading error (system interaction error).

Error of technique.

Instruments Errors

b. Illegitimate errors

Chaotic errors.

c. Random errors

Environmental error

Errors Due to Calibration:

Any instrument has to be calibrated before it is put to use. Calibration is a process of giving a known input to the measurement system and taking necessary actions to see that the output of the measurement system matches with its input.

If the instrument is not calibrated properly, it will show reading with a higher degree of error. This is called as calibration error.

Calibration errors are fixed errors as they have been introduced into the measurement system because of improper calibration.

Human Errors:

There are two human errors namely the observation errors and the operational errors.

There is a saying that "Instruments are better than the people who use them". Even if a good instrument is available, errors are introduced due to the user.

Observation errors are due to improper observation made by the user of the instrument.

Example (1): If the graduations on the scale are very close, the observer may read incorrectly.

Example (2): Not observing continuously when it is necessary to do so.

Loading Errors (System Interaction Errors):

The measuring instrument always takes energy from the signal source (measured medium) and due to this the signal source is always altered by the act of measurement. This effect is called as loading.

As the measured quantity loses energy due to the act of measurement, error is introduced. This is called as loading error.

Example (1): When a thermometer is introduced, it alters the thermal capacity of the system and heat leakage takes place. Due to this error occurs.

Example (2): Reading of a hand tachometer will vary depending on the pressure with which it is pressed on the shaft.

Error of technique:

Improper use of exact technique for executing an operation leads to this type of error.

Instruments Errors:

The accuracy of an instrument is affected due to limitations in its design and construction.

Example (1): The components of an instrument may be assembled incorrectly and due to this an error may occur. This error does not vary with time and can be corrected.

Example (2): An improper material may be selected for the instrument causing it to wear quickly or creating friction, thus introducing an error.

b. Illegitimate Error

Chaotic Error

Errors induced by random disturbances such as vibrations, noises, shocks etc., of sufficient magnitude tend to affect the test information. Such errors are called as chaotic errors.

Due to such random disturbances, the instrument cannot measure the physical quantity properly and more over there will be information lose during signal transmission, this is called as transmission error.

Uncertainty and Random Error

Uncertainty and random errors are indicated when repeated measurements of the same quantity result in differing values. The magnitude and direction of these errors are not known and as such are considered indeterminate.

They are caused by such effects as friction, spring hysteresis, noise, and other phenomena. The contributing factors are any random changes in input signal, combined with noise and drift in the signal conditioner.

Such errors occur more in dynamic data analysis. The uncertainty is expressed as the average deviation, probable error, or statistical deviation. The error value is estimated as the amount by which the observed or calculated value departs from the true value.

Environmental Error

Any instrument is manufactured and calibrated at one place and is put in use at some other place where the environmental conditions such as pressure, temperature, humidity etc., are different.

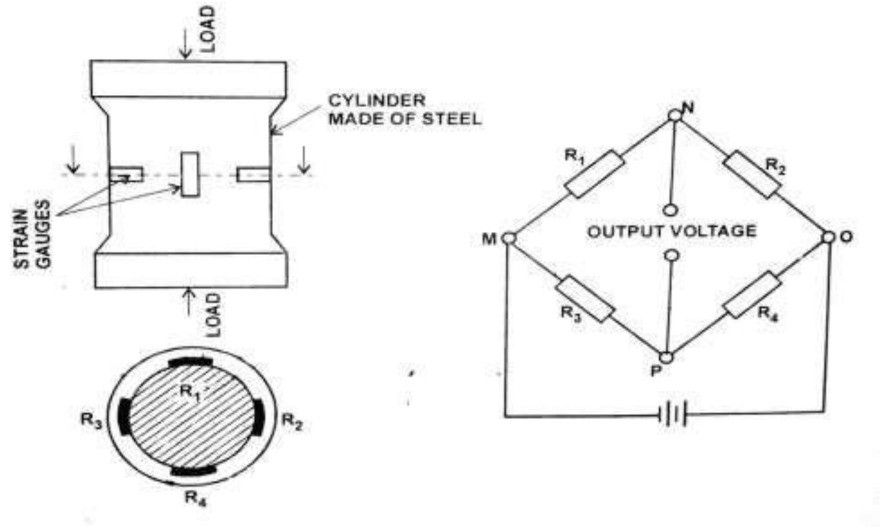
This change in environment influences the readings of the instrument. This change in reading of the instrument due to environmental changes is called as environmental error.

Example (1): If a mercury - in glass thermometer is located at a place where the air pressure is high, the air pressure acts on the walls of the thermometer causing the mercury to rise even without a change in temperature.

Example (2): A bourdon-tube pressure gauge has a link-sector-pinion arrangement. The link may expand if the environmental temperature increases, causing an error.

13. Describe briefly about strain gauges and its types.

When a steel cylinder is subjected to a force, it is tends to change in dimension. On this cylinder if strain gauge is bonded, the strain gauge also is stretched or compressed, causing a change in its length and the diameter. This change in dimension of the strain gauge causes its resistance to change. This change in resistance (or output voltage) of the strain gauge becomes a measure of the applied force.



Description:

The main parts of the strain gauge load ell are as follows:

Cylinder made of steel on which four identical strain gauge are mounted.

Out of the four strain gauges, two of them (R1 and R4) are mounted along the direction of the applied load.

The other two strain gauge (R2 and R4 horizontal gauges) are mounted circumferentially at right angles to gauges R1 and R4.

The four gauges are connected to the four limbs of a wheat stone bridge.

Operation:

When there is no load on the steel cylinder, all the four gauges will have the same resistance. As the terminals N and P are at the same potential, the Wheatstone bridge is balanced and hence the output voltage will be zero.

Now the load to be measured is applied on the steel cylinder. Due to this, the vertical gauges will undergo compression and hence there will be decrease in resistance.

The horizontal gauges will undergo tension and there will be increase in resistance, thus when strained, the resistance of the various gauges change.

Now the terminals N and P will be at different potential and change in output voltage due to the applied load becomes a measure of the applied load when calibrated.

14. Discuss in details

Hydraulic load cells

When a force is applied on a liquid medium contained in a confined space, the pressure of the liquid increases. This increase in pressure of the liquid is proportional to the applied force. Hence a measure of the increase in pressure of the liquid becomes a measure of the applied force when calibrated.

Description

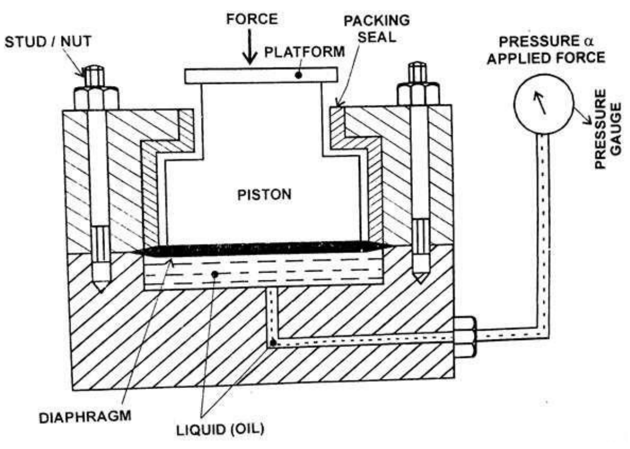
The main parts of a hydraulic are as follows:

A diaphragm.

A piston with a loading platform place on top of the diaphragm.

A liquid medium which is under a pre-loaded pressure is on the other side of the diaphragm.

A pressure gauge (Bourdon tube type) connected to the liquid medium.



Operation:

The force to be measured is applied to the piston.

The applied force moves the piston down wards and deflects the diaphragm and this deflection of the diaphragm increases the pressure in the liquid medium (oil).

This increase in pressure of the liquid medium is [proportional to the applied force. This increase in pressure is measured by the pressure gauge which is connected to the liquid medium.

The pressure is calibrated in force units and hence the indication in the pressure gauge becomes a measure of the force applied on the piston.

b. **Pneumatic load cell**

Basic principle

If a force is applied to one side of a diaphragm and air pressure is applied to the other side, some particular value of pressure will be necessary to exactly balance the force. This pressure is proportional to the applied force.

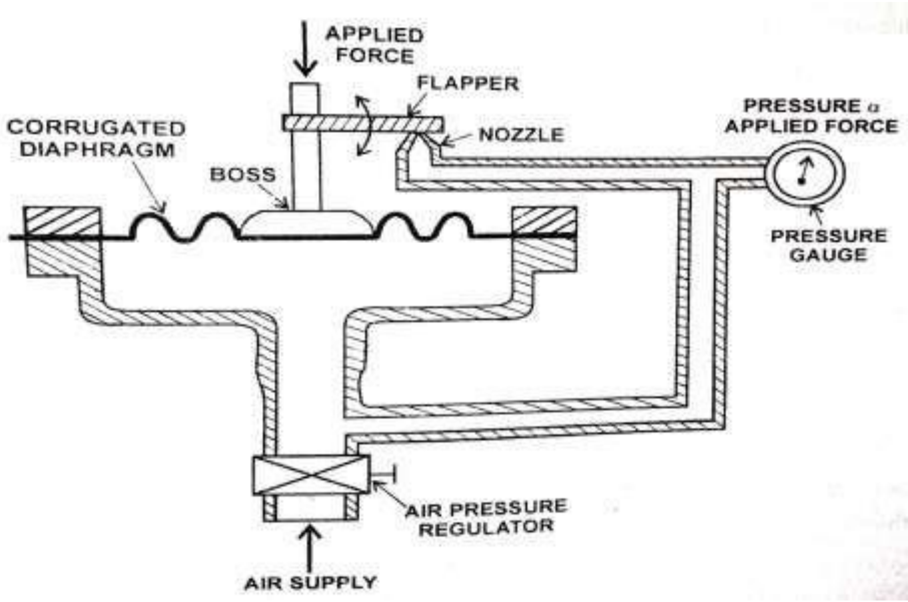
Description:

The main parts of a pneumatic load cell are as follows:

A corrugated diaphragm with its top surface attached with arrangements to apply force.

An air supply regulator, nozzle and a pressure gauge arranged as shown in figure.

A flapper arranged above the nozzle as shown in figure.



Operation:

The force to be measured is applied to the top side of the diaphragm. Due to thisforce, the diaphragm deflects and causes the flapper to shut-off nozzle opening.

Now an air supply is provided at the bottom of the diaphragm. As the flapper closes the nozzle opening, a back pressure results underneath the diaphragm.

This back pressure acts on the diaphragm producing an upward force. Air pressure is regulated until the diaphragm returns to the pre-loaded position which is indicated by air which comes out if the nozzle.

At this stage, the corresponding pressure indicated by the pressure gauge becomes a measure of the applied force when calibrated.

15. Explain and classify the dynamic pressure sources.

Dynamic characteristics refer to the performance of the instrument when the input variable is changing rapidly with time. For example, human eye cannot detect any event whose duration is more than one-tenth of a second; thus the dynamic performance of human eye cannot be said to be very satisfactory. The dynamic performance of an instrument is normally expressed by a differential equation relating the input and output quantities. It is always convenient to express the input-output dynamic characteristics in form of a linear differential equation. So, often a nonlinear mathematical model is linearised and expressed in the form

$$a_n \frac{d^n x_0}{dt^n} + a_{n-1} \frac{d^{n-1} x_0}{dt^{n-1}} + \cdots + a_1 \frac{dx_0}{dt} + a_0 x_0 = b_m \frac{d^m x_i}{dt^m} + b_{m-1} \frac{d^{m-1} x_i}{dt^{m-1}} + \cdots + b_1 \frac{dx_i}{dt} + b_0 x_i$$

where xi and x0 are the input and the output variables respectively. The above expression can also be expressed in terms of a transfer function, as:

$$G(s) = \frac{x_0(s)}{x_i(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} \cdots + b_1 s + b_0}{a_n s^n + b_{n-1} s^{n-1} \cdots + a_1 s + a_0}$$

Normally $m<n$ an n is called the order of the system. Commonly available sensor characteristics can usually be approximated as either *zero-th order*, *first order* or *second order* dynamics. Here are few such examples:

Potentiometer

Displacement sensors using potentiometric principle (Fig.4) have no energy storing elements. The output voltage e_o can be related with the input displacement x_i by an algebraic equation:

$$e_o(t)x_t = Ex_i(t); \quad or, \quad \frac{e_o(s)}{x_i(s)} = \frac{E}{x_t} = constant \qquad (8)$$

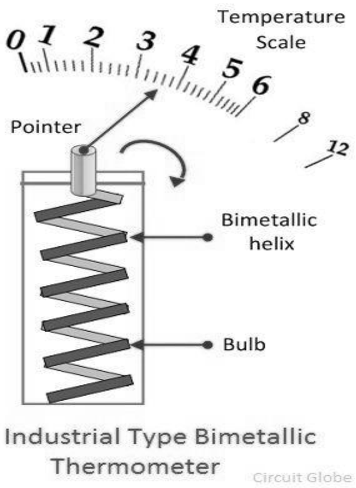
where x_t is the total length of the potentiometer and E is the excitation voltage.. So, it can be termed as a *zeroth order system*.

16. Discuss the temperature measurements

a) Bi-metallic thermometer

Helical Types Bimetallic Strip –

The helix type bimetallic strip is mostly used for industrial applications. In this thermometer, the helix shape strip is used for measuring the temperature. The free end of the strip is connected to the pointer. The deflection of the strip shows the variation of temperature.



Advantages

The thermometer is simple in construction, robust and less expensive.

Disadvantages

The thermometer gives the less accurate result while measuring the low temperature.

Applications of Bimetallic Thermometer

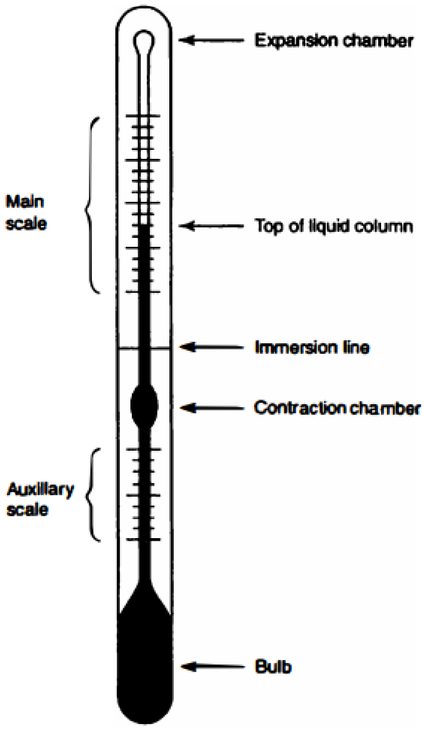
The bimetallic thermometer is used in household devices likes oven, air conditioner, and in industrial apparatus like refineries, hot wires, heater, tempering tanks etc. for measuring the temperature.

b) Pressure thermometers

Liquid in glass thermometers

The ordinary thermometer is an example of the liquid-in-glass type as shown in figure. Its essential elements are a relatively large bulb at the lower end, a capillary tube with scale, and a liquid filling both the bulb and a ponion of the capillary. In addition, an expansion chamber is generally incorporated at the upper end to serve as a safety reservoir when the intended temperature range is exceeded.

As the temperature is raised, the greater expansion of the liquid compared with that of the glass causes it to rise in the capillary or stem of the thermometer, and the height of rise is used as a measure of the temperature. The volume enclosed in the stem above the liquid may either contain a vacuum or be filled with air or another gas. For the higher temperature ranges, an inen gas at a carefully controlled initial pressure is introduced in this volume, thereby raising the boiling point of the liquid and increasing the total useful range. In addition, it is claimed that such pressure minimizes the potential for column separation.



High-grade liquid-in-glass thermometers may include several additional features. An immersion line may be inscribed on the thermometer to indicate the depth to which it should be submerged into the measured environment. A contraction chamber may be provided to shorten the overall length of capillary needed and to prevent bubbles from being formed into the bulb when the thermometer is cooled. Finally, an auxiliary scale may be provided for checking calibration points outside the main range of the thermometer, such as 0^oC or 100^oc.

17. Discuss about Vibrational exciter systems.

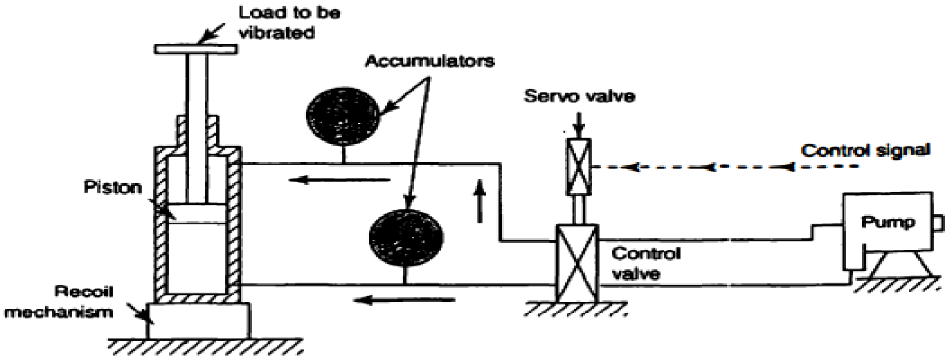


FIGURE 11: Block diagram of a hydraulically operated shaker.

Frequency Range

The upper frequency ranges are available only through use of the electromagnetic shaker. In general, the larger the force capacity of the electromagnetic exciter, the lower its upper frequency will be. However, even the 40,000-lbf (180-kN) shaker listed in Table 2 boasts an upper useful frequency of 2000 Hz. To attain this value with a mechanical exciter would require speeds of 120,000 rpm. The maximum frequency available from the smaller mechanical units is limited to approximately 120 Hz (7200 rpm) and for the larger machines to 60 Hz (3600 rpm). Hydraulic units are presently limited to about 2000 Hz.

Force Limitations

Electromagnetic shakers have been built with maximum vector force ratings of 40,000 lbf (180 kN). Variable-frequency power sources for shakers of this type and size are very expensive. Within the frequency limitations of mechanical and hydraulic systems, corresponding or higher force capacities may be obtained at lower costs by hydraulic shakers. Careful design of mechanical and hydraulic types is required, however, or maintenance costs become an important factor. Mechanical shakers are particularly susceptible to bearing and gear failures, whereas valve and packing problems are inherent in the hydraulic ones.

Maximum Excursion

One inch, or slightly more, may be considered the upper limit of peak-to-peak displacement for the electromagnetic exciter. Mechanical types may provide displacements as great as 5 or 6 in.; however, total excursions as great as 18 in. have been provided by the hydraulic-type exciter.

Magnetic Fields

Because the electromagnetic shaker requires a relatively intense fixed magnetic field, special precautions are sometimes required in testing certain items such as solenoids or relays, or any device in which induced voltages may be a problem. Although the flux is rather completely restricted to the magnetic field structure, relatively high stray flux is nevertheless present in the immediate vicinity of the shaker. Operation of items sensitive to magnetic fields may therefore be affected. Degaussing coils are sometimes used around the table to reduce flux level.

Nonsinusoidal Excitation

Shaker head motions may be sinusoidal or complex, periodic or completely random. Although sinusoidal motion is by far the most common, other waveforms and random motions are sometimes specified [12]. In this area, the electromagnetic shaker enjoys almost exclusive franchise. Although the hydraulic type may produce nonharmonic motion, precise control of a complex waveform is not easy. Here again, future development of valving may alter the situation.

The voice coil of the ordinary loudspeaker normally produces a complex random motion, depending on the sound to be reproduced. Complex random shaker head motions are obtained in essentially the same manner. Instead of using a fixed-frequency harmonic oscillator as the signal source, either a strictly random or a predetermined random signal source is used. Electronic *noise* sources are available, or a record of the motion of the actual end use of the device may be recorded on magnetic tape and used as the signal source for driving the shaker. As an example, electronic gear may be subjected to combat-vehicle

18. Explain the Fast Fourier Transformer concepts.

A fast Fourier transform (FFT) is an algorithm that computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT). Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa. The DFT is obtained by decomposing a sequence of values into components of different frequencies.[1]This operation is useful in many fields, but computing it directly from the definition is often too slow to be practical. An FFT rapidly computes such transformations by factorizing the DFT matrix into a product of sparse (mostly zero) factors. As a result, it manages to reduce the complexity of computing the DFT from which arises if one simply applies the definition of DFT, to where is the data size. The difference in speed can be enormous, especially for long data sets where N may be in the thousands or millions. In the presence of round-off error, many FFT algorithms are much more accurate than evaluating the DFT definition directly or indirectly. There are many different FFT algorithms based on a wide range of published theories, from simple complex-number arithmetic to group theory and number theory.

Time-based representation (above) and frequency-based representation (below) of the same signal, where the lower representation can be obtained from the upper one by Fourier transformation.

Fast Fourier transforms are widely used for applications in engineering, music, science, and mathematics.

The Fast Fourier Transform (commonly abbreviated as FFT) is a fast algorithm for computing the discrete Fourier transform of a sequence. The purpose of this project is to investigate some of the mathematics behind the FFT, as well as the closely related discrete sine and cosine transforms. I will produce a small library of MATLAB code which implements the algorithms discussed, and I will also look into two real-world applications of the FFT: solving partial differential equations and JPEG compression.

I began by studying the Fourier transform and the discrete Fourier transform, by reading the textbook [1], which has a chapter dedicated to the FFT and related concepts. From here I gained an understanding of how the discrete Fourier transform is related to the continuous Fourier transform, as well as how the FFT works.

The Fourier transform is an integral transform given by the formula

$$\mathcal{F}\{f(t)\} = \hat{f}(k) = \int_{-\infty}^{\infty} e^{-2\pi ikt} f(t)\,\mathrm{d}t.$$

It takes the function $f(t)$ as input and outputs the function $\hat{f}(k)$. We usually think of f as a function of time t and $\hat{f}$ as a function of frequency k. The Fourier transform has various properties which allow for simplification of ODEs and PDEs. For example, if $f^{(n)}$ denotes the nth derivative of f, then $\mathcal{F}\{f^{(n)}(t)\}(k) = (2\pi ik)^n \mathcal{F}\{f(t)\}(k) = (2\pi ik)^n \hat{f}(k)$ [2].

The discrete Fourier transform (DFT) of an array of N complex numbers $f_0, f_1, \ldots, f_{N-1}$ is another array of N complex numbers $F_0, F_1, \ldots, F_{N-1}$, defined by

$$F_n = \sum_{j=0}^{N-1} f_j e^{-2\pi inj/N}.$$

The DFT can provide an approximation to the continuous Fourier transform of a function f. Suppose we take N samples $f_j = f(t_j)$, where $t_j = jh$, $j = 0, 1, \ldots, N-1$, where h is the sampling interval. Then we can estimate that $\hat{f}(k_n) \approx hF_n$ at the frequencies $k_n = \frac{n}{Nh}$, $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}$.

Taking inspiration from the algorithms at [4] and [5], I implemented the FFT in two slightly different ways using the programming language MATLAB. I then used a MATLAB script to compare the runtimes of both algorithms.

Given an ODE $\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$ and an initial condition $y(t_0) = y_0$, we can approximate the solution y with Euler's method. Fix a step size h, then let $t_n = t_0 + hn$, for $n = 0, 1, \ldots, N$. Then perform the recurrence relation $y_{n+1} = y_n + hf(t_n, y_n)$ for $n = 0, 1, \ldots, N$, to obtain the approximation $y(t_n) \approx y_n$. This is the simplest method for numerically solving initial value problems, but with a small enough step size, it can be very accurate [6].

One final technique, which is the central idea behind the compression of JPEG image files, is the discrete cosine transform. The DCT is similar to the DFT, but one main difference is that it uses real numbers only. The reason the DCT is well-suited to compression is because a signal can be reconstructed with reasonable accuracy from just a few low-frequency components of its DCT.

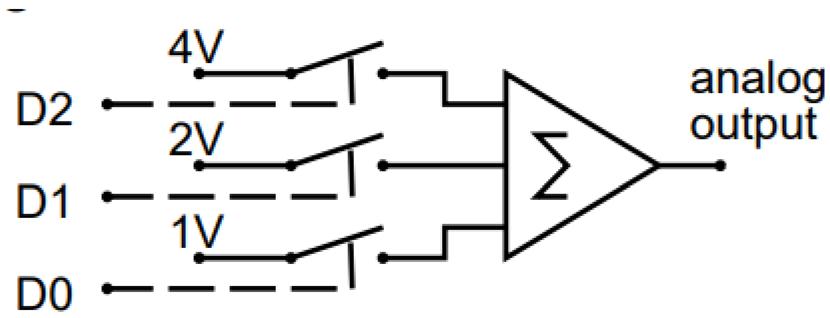
19. Explain briefly with circuit diagrams

a) DAC

Digital-to-Analog Conversion

A digital-to-analog (D/A) converter (DAC) is a chip or circuit that converts a digital number into an analog voltage or current. D/As are used to control devices that require a range of control voltages or currents such as electro-acoustic transducers (speakers), some types of variable-speed motors, and many other applications where an analog signal output is required. The most common application is to [re-]create waveforms from digital signals – for example in CD players.

D/A Converters A D/A converter can be visualized as a circuit that adds up a number of voltages under the control of a digital signal. Each voltage can be turned on and off by an electronic switch which is controlled by the digital input. The circuit below shows a 3-bit D/A consisting of a summing amplifier fed by three different voltages. Depending on which switches are closed, the output can range from 0 to 7 volts. By using digital signals to control the switches we can build a circuit whose output voltage is proportional to the digital value.



D/A Converter Specifications

Many different Digital-to-Analog converters are commercially available, both as chips and as subsystems (modules, boards, etc). To select the right D/A converter it is necessary to understand D/A specifications. The most important specs are resolution and settling time. In some applications other specifications such as slew rate, linearity, monotonicity may also be important. There are also various types of digital and analog interfaces. Since different manufacturers use different definitions for some of these specifications it is important to check each manufacturer's definitions when comparing devices. The resolution (output step size) is given by the output voltage range divided by the number of possible output levels. An N-bit

DAC can output 2N different levels in 2^N-1 steps. N ranges from 6 (for simple high-speed converters) up to 20 or more (for precision instrumentation).

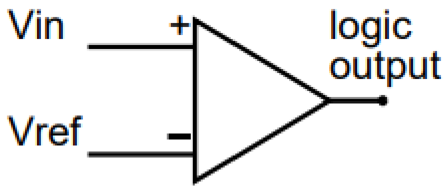
b) ADC

Analog to Digital Converters

A controller often has to measure a physical quantity, for example temperature, pressure, force, etc. A sensor, often called a transducer, is used to convert this physical quantity into an electrical signal (current or voltage).

This electrical signal must then be converted into a binary number so that the digital controller can use it. An analog to digital (A/D) converter (ADC) performs this function. Comparators

The simplest type of ADC is a comparator. As its name indicates, a comparator compares two analog inputs (say vin and vref) and outputs a logic signal which is high if Vin is greater than Vref or low otherwise. Comparators are available as ICs, often with two or four units in one IC.

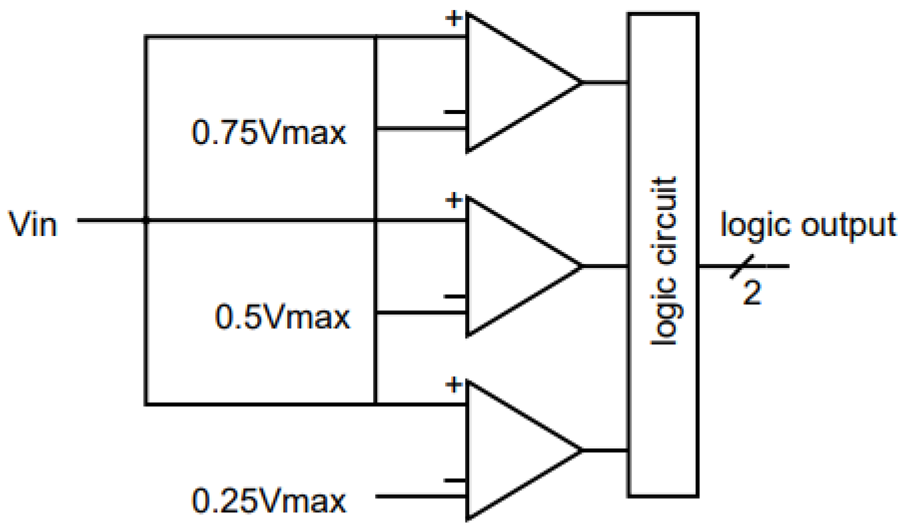


c)

The comparator can be considered to be a one-bit ADC since it's output tells us whether the input voltage is above or below the reference voltage. Although a comparator does not offer much precision, it is often sufficient since for many applications it's only necessary to determine whether some quantity is above or below some threshold. Comparators also form the basis of other types of A/D converters as will be described below.

Types of A/D Converters Flash Converters A simple way to get better (more bits of) resolution is to use more comparators. As shown below for a 2-bit flash converter we can use 2^N-1 comparators, supplying them with reference voltages that are equally spaced over the desired conversion range. The other comparator inputs are connected to the input signal.

All of the digital outputs connected to reference voltages below the input signal will be true and all of the outputs with reference signals above the input signal level will be false. The output logic circuit converts these 2^N-1 binary values into an N-bit number.



20. Briefly discuss the representation methods of experimental data.

Linear Least Squares Regression

In the previous section we defined statistical regression as an activity that produces a model that "best describes the relationship" between y and the independent variables. Conventionally, the criterion we use to define "best" derives from the method of maximum likelihood. In words, the method of maximum likelihood seeks the values of the parameters that maximize the probability of having obtained the set of observations in hand. If we assume that the errors in y are independent and normally distributed, the method of maximum likelihood leads to a goodness-of-fit parameter, X^2 ("chi-squared"), which is defined as

$$X^2 \equiv \sum_{i=1}^{n}\left(\frac{y_i - f(x_{1i}, x_{2i}, \ldots, x_{li}; a_1, a_2, \ldots, a_m)}{\sigma_i}\right)^2$$

where σi is the standard deviation associated with the ith value of y. The "best" values of the parameters are those that minimize X^2 . It is quite often the case that we do not know σi. If we make a further assumption that the observations of y are identically distributed, i.e. not only do all values have the same underlying error structure (in this case a normal distribution) but they also have the same magnitude of error, then the value of σi is the same for all i. Taken together, the three assumptions are stated concisely as the errors are IID and normally distributed. With this assumption, X^2 becomes

$$X^2 = \frac{1}{\sigma^2}\sum_{i=1}^{n}(y_i - f(x_{1i}, x_{2i}, \ldots, x_{li}; a_1, a_2, \ldots, a_m))^2$$

where σ is the common standard deviation in y. Minimizing X^2 is now a matter of finding the values of $\{a_1, a_2, \ldots, a_m\}$ that minimize the summation on the right hand side of equation. This form of statistical regression is called least squares regression.

Experimentalists should be aware of the times when it is not appropriate to assume that all the measurements have the same standard deviation. If one is using an instrument for which the error is proportional to the magnitude of the measurement then the errors will be small for measurements of smaller values and vice versa. Similarly, if the data to be used in the regression were obtained via different measurement techniques then the errors can be substantially different.

For example, if low values of aqueous concentration are measured using a calibration curve designed specifically for a low concentration range, and high values of aqueous concentration are measured using a calibration curve designed for a high concentration range, then it is quite likely that they two sets of observations will have different measurement errors.

The errors may be constant in a relative sense (e.g. the % error), but the absolute errors may be very different. After the regression has been performed, one can test the assumption of common errors by examining a plot of the residuals (i.e. estimated errors) versus the x variable. The residuals should have a random appearance and not appear to be related to the value of x. If the experimentalist has good reason to be concerned about the lack of a common error in the observations of y then the extra effort should be taken to estimate the errors in the observations. In this case, regression should be performed by minimizing the X^2 function in equation. This is a form of weighted least squares regression.

If the function $f(x_{1i}, x_{2i}, \ldots, x_{li}; a_1, a_2, \ldots, a_m)$ i is linear in the parameters then the summation in equation can be analytically differentiated with respect to each of the parameters. Setting these differentials equal to zero produces m linear equations which can be solved simultaneously for the values of the parameters that minimize X^2 . This form of statistical regression is called linear least squares regression.